The Coupled Motions of Piano Strings

Most of the notes on a piano are sounded by two or three strings. The strings are not tuned to precisely the same frequency, a fact that contributes in unexpected ways to the tone of the instrument

by Gabriel Weinreich

In 1709 the Italian harpsichord maker Bartolommeo Cristofori built the prototype of the modern piano in which hammers strike the strings. Since that time pianos have always been made with more than one string corresponding to most notes. The keyboard of the modern Steinway Model B has 88 keys, with the 68 highest ones setting in motion triplets of strings and the 20 lowest ones setting in motion pairs of strings or single strings. Although the tripling and pairing of strings was introduced in keyboard instruments in the middle of the 17th century to increase the volume of the sound, recent acoustical work demonstrates that in the piano, at least, the tripling and pairing also affects the quality of the sound. The work reveals that when the strings that constitute a single triplet or pair are made to sound separately, they differ slightly in frequency in a seemingly random way, even after the best piano technicians have tuned them. Remarkably, listeners have turned out to prefer the sound made by a key when there was a small discrepancy among the individual frequencies to the sound made by a key when there was no discrepancy.

My own work at the University of Michigan has focused on how these small "mistunings," or frequency discrepancies, contribute to the sound of the piano. I have also investigated other ways the tripling and pairing of strings has changed the sound from what it would have been if pianos had been built with only one string per note. Out of my work and that of other investigators has emerged a detailed picture of how the sound at first decays rapidly (the "prompt sound") and then decays slowly (the "aftersound").

When a piano key is depressed, a felt-covered hammer strikes the corresponding unison group of three or two strings. At the same time the block of soft felt called a damper is lifted from the strings so that they can vibrate freely. Releasing the key actuates the damper that stops the strings from vibrating. The sound will also cease if the vibrations are allowed to die away to inaudibility of their own accord.

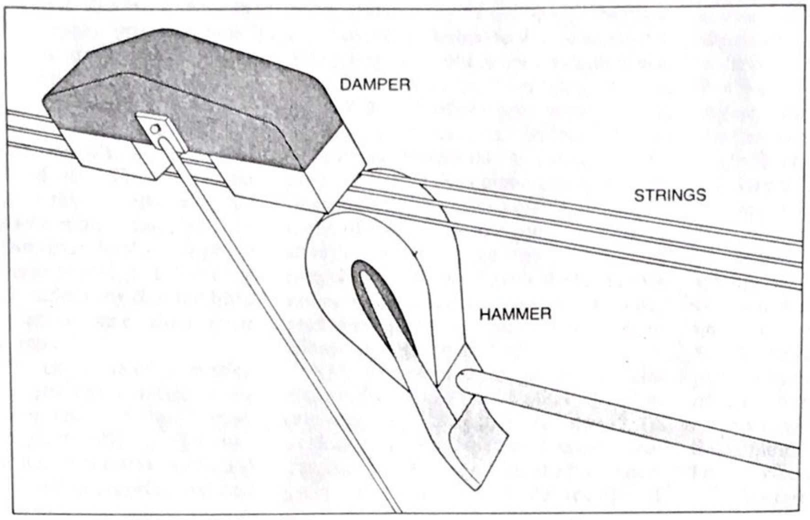
Every musical sound ultimately originates with internal vibrations of the instrument. In wind instruments such as the flute and instruments with bowed strings such as the violin the vibrations are "sustained": energy is fed into them in the course of each oscillation. In the piano, on the other hand, the vibrations are "free": no energy is fed in after the initial impact of the hammer. As a result the musical characteristics of the piano depend mainly on how the energy in the strings is dissipated.

The energy is dissipated as a result of frictional forces that manifest themselves in various parts of the piano. When a string vibrates with small amplitudes, the energy dissipation is "linear": the rate at which the string loses energy is proportional to the amount of energy it contains. In such a linear system the decay of the vibration is exponential: equal fractions of energy are dissipated in equal intervals of time. The radioactive decay of carbon 14 is a familiar example of a linear system that is exactly analogous to the vibrational decay of a piano string. The rate at which a material loses carbon 14 is proportional to the amount of carbon 14 it contains. The fact that half of the carbon 14 decays every 5,730 years means the rate of decay is exponential, just as it is for the energy of a piano string.

The ratio of the pressure amplitudes of two sounds is measured in decibels, with every 20 decibels corresponding to a change in the sound pressure by a factor of 10. Hence a decay of 40 decibels corresponds to a drop in the sound pressure by two factors of 10, or a factor of 100, and a decay of 10 decibels corresponds to a drop in the sound pressure by a factor of $\sqrt{10}$, or a factor of 3.16. When the sound pressure is specified by giving its ratio (in decibels) to some standard fixed value, it is called the sound-pressure level. One advantage of such an approach is that for exponential decay a plot of the sound-pressure level as a function of time turns out to be a straight line. Sound pressure is a physical quantity that is similar to loudness but not exactly the same. Sound pressure is strictly a physical phenomenon, whereas loudness is a psychophysical phenomenon that depends not only on the physical properties of the sound but also on how the ear and the brain respond to them. For example, a doubling of sound pressure would not necessarily be perceived as a doubling of loudness. For qualitative purposes, however, it is safe to think of sound pressure and loudness as being almost the same thing.

The decay of the sound of a piano string (when the other strings in its unison group are not allowed to vibrate) is actually more complicated than the picture of a single straight line suggests. The plot of sound-pressure level as a function of time turns out to be not one straight line but a kinked line consisting of two straight segments, the first starting at a high level and decaying quickly,

EXPLODED VIEW OF STEINWAY MODEL B PIANO in the illustration on the opposite page shows the relations of the main components. The keyboard has 88 keys that are divided into seven and a third octaves. The 68 highest keys (*on the right*) set in motion triplets of strings, and the 20 lowest ones (*on the left*) set in motion pairs of strings or single strings. Connected to the keyboard is the action, which includes hammers and dampers that determine the string motions. When a key is depressed, a hammer sets the strings vibrating. The strings cross a wood bridge that transmits the vibrations to the soundboard, from which the vibrations are radiated into the air. Sound is also radiated to a lesser degree from other parts of a piano.



HAMMER HITS THE STRINGS that correspond to one note with the same strength and at the same time. Because the hammer strikes the strings in the vertical direction, they move mostly in that direction. They also move, however, a little in the horizontal direction. This motion could be caused by small irregularities in the face of the hammer or the position of the strings.

and the second taking over at a lower level and decaying slowly [*see upper illustration on opposite page*]. This means that although the sound is always decaying exponentially, the rate of exponential decay changes fairly abruptly at one point. The initial fast decay characterizes the prompt sound and the final slow decay characterizes the aftersound. The presence of these two kinds of sound is a characteristic feature of the piano tone. The prompt sound has something of a "ping," similar to that of the xylophone, but whereas after a few seconds a xylophone becomes silent, a piano is still singing away. It is this singing that constitutes the aftersound and enables sustained melodies to be played on the piano that cannot be played on the xylophone.

The presence of prompt sound and aftersound is not the result of nonlinearity: proportionately larger frictional forces at higher sound pressures causing a higher rate of decay at the beginning. Such nonlinearities are commonplace in many other physical systems. For example, a pendulum that is subject to only a small amount of friction at its pivot point will be slowed by air friction that is nonlinear. At large amplitudes the pendulum agitates the air violently, whereas at small amplitudes it moves the air smoothly. The air friction is nonlinear because the turbulent air at large amplitudes exerts a proportionately larger force on the pendulum than the smoothly moving air at small amplitudes.

That a similar phenomenon is not the cause of the aftersound of a piano can be demonstrated experimentally. If the frictional forces were nonlinear, the break from fast decay to slow decay would take place at the same amplitude regardless of the initial displacement of the string. The initial displacement would affect only the length of the prompt sound; the larger the displacement, the longer the prompt sound. This means that plots of sound-pressure level as a function of time made for various initial displacements would be horizontal translations of one another [*see top illustration on page 122*].

In fact the break from fast decay to slow decay actually takes place at different sound pressures but at the same number of seconds after the initial displacement. In other words, the prompt sound lasts for the same amount of time regardless of how much the string is initially displaced. The sound-pressure plots are simply vertical translations of one another, not the horizontal translations indicative of nonlinear friction. In a linear system such as a piano string the amplitude of the motion can be uniformly enlarged or diminished without changing the quality of the motion.

What is it, though, that causes the change in the rate of decay? It turns out that for a single string the change results from the existence of two polarizations of vibration in the string..I shall refer to them as vertical and horizontal, corresponding to the actual directions in a grand piano. Other motions, including circular and elliptical ones, can be thought of as superpositions of the two basic polarizations. Since the hammer hits the string in the vertical direction, it may seem strange that the string acquires any horizontal motion at all. Indeed, initially the string is moving mostly in the vertical direction. What little horizontal motion there is initially could be the result of small irregularities in the face of the hammer or the position of the string. In other words, the slight horizontal motion comes from the fact that the hammer does not strike the string precisely in the vertical direction.

I used a sensitive electronic probe to separately measure the vertical and horizontal motions of a single string. My measurements showed that each separate polarization decays exponentially but that the vertical one decays much faster [*see upper illustration on opposite page*]. This means that although the vertical motion is initially much stronger than the horizontal one (probably by at least a factor of 10), the horizontal motion eventually comes to be dominant. Hence the vertical motion gives rise to the short-lived prompt sound and the horizontal motion gives rise to the long-lived aftersound.

Why does the rate of decay differ for the two kinds of motion? The answer depends on the ways the string can lose energy. When the string loses vibrational energy, the energy can be lost to heat inside the string (internal friction), to the motion of the adjacent air (viscosity and sound radiation) or to the string's supports. In the piano this last mechanism is the dominant one. The keyboard end of the string is fixed against an iron frame, whereas the far end goes over a wood "bridge" that is glued to the soundboard. The bridge is not totally rigid because its function is to make the soundboard vibrate synchronously with the string. Most of the sound is radiated into the air from the soundboard, although it is also radiated to a lesser degree from other parts of the piano. The vertical motion of the soundboard turns out to have much more "give" than the horizontal motion, and so energy is easily transferred from the vertical motion of the string to the vertical motion of the soundboard. This accounts for the faster decay of the string's vertical motion, which is responsible for the prompt sound. In due course I shall modify this simple picture, because the mere presence of "give" does not necessarily lead to energy loss.

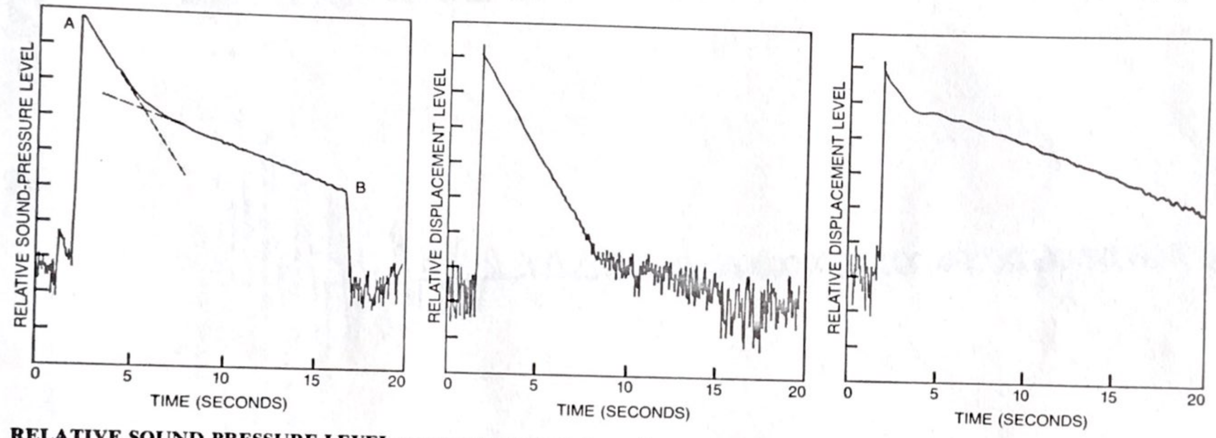
Acoustical physicists have not yet investigated in detail how the horizontal vibrations radiate. I performed a simple experiment to see whether or not the horizontal sound waves and the vertical ones emanate from the same source in the piano. (Strictly speaking, the sound waves themselves are not horizontal and vertical. Here these terms refer to the string polarizations to which the sound waves correspond.) If they emanate from different sources, then when they are of comparable strength, they interfere, or combine, to form a sound wave whose amplitude at any given point in the room equals the sum of the amplitudes of the component waves.

Where the waves are exactly in phase (crests coinciding and troughs coinciding) the amplitude is increased, and where they are exactly out of phase (crests coinciding with troughs) the amplitude is decreased. To see if I could detect this interference phenomenon I put a microphone at different places around a piano and examined the behavior of the sound-pressure level during the transition from prompt sound to aftersound. Sure enough, at the time of transition I found the points where the component waves reinforced each other and the points where they canceled each other. This means that the relative phase of the component waves is different at different points, indicating that the horizontal and vertical waves emanate from different "antennas."

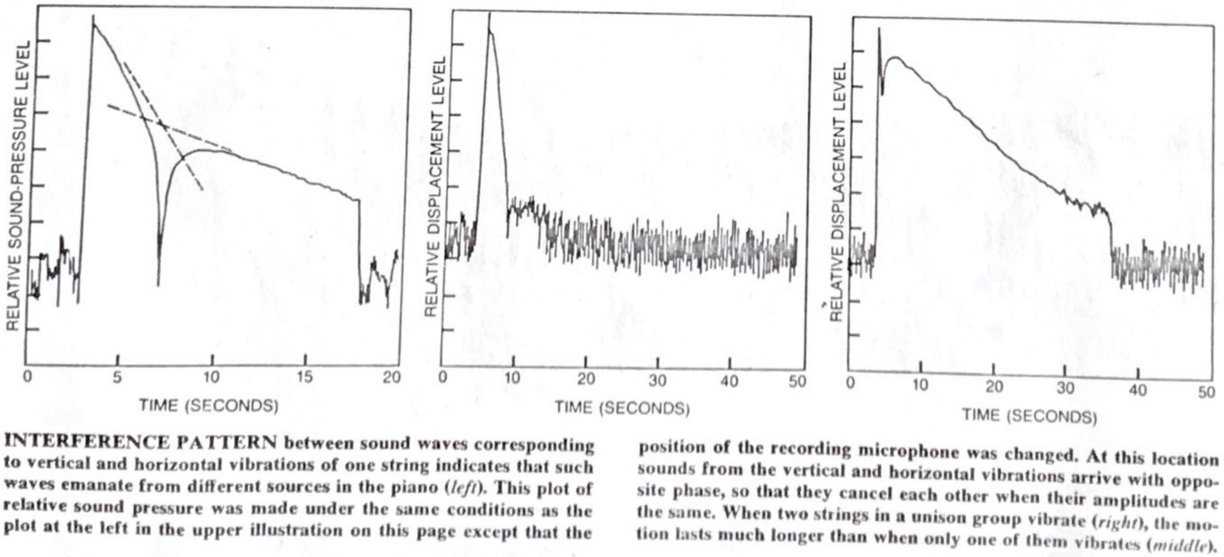
The existence of horizontal motion constitutes only one contribution to the aftersound, a contribution that is independent of the fact that the strings come in unison groups of three and two. Thus an aftersound due to horizontal motion would still be heard if pianos were built with only one string per note. The tripling and pairing of the strings, however, also contributes to the aftersound. The strings that make up a unison group cross the bridge close to one another. As a result their motions are coupled: when one string vibrates, the bridge vibrates with it and transmits the motion to the other strings in the unison group. Experiments show that the vertical displacement of one of these coupled vibrating strings sometimes decays at the slow rate characteristic of aftersound. Hence uncoupled horizontal motion and coupled vertical motion independently contribute to the aftersound.

Why does the coupled motion decrease the decay rate rather than increasing it or leaving it the same? To explain this phenomenon I must introduce the fact that the rate at which energy is dissipated through the motion of the bridge is determined not by whether the bridge can move but whether it does move. Specifically, if two strings cross the bridge at the same place and vibrate with the same frequency and amplitude but with opposite phase (one string going up while the other is going down), the net force on the bridge will be zero. Therefore the bridge will not move at all. To each string separately the bridge will seem to be completely rigid. If, on the other hand, the strings vibrate with the same phase (as well as the same frequency and amplitude), the bridge motion will be double what it would be if only one string were vibrating. Hence the decay rate also doubles. The same principles that govern the dissipation of energy in two strings govern it in three strings, although in the latter case the difficulty of following three phase factors can obscure what is going on.

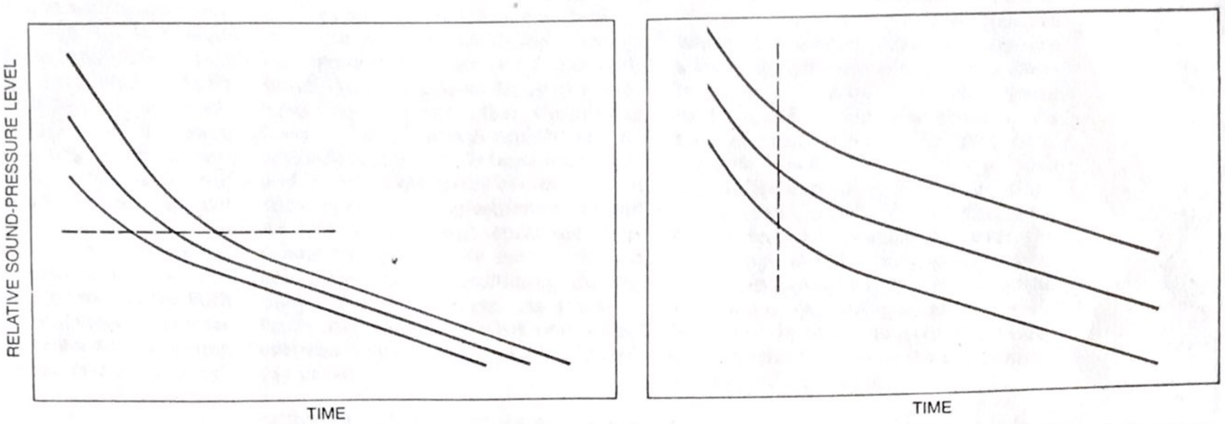
In most situations in acoustical physics



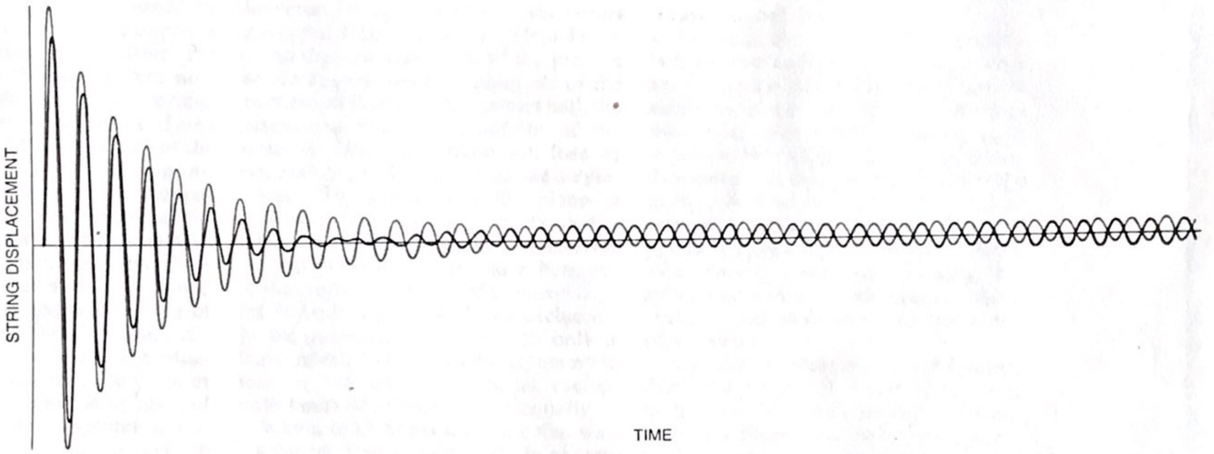
RELATIVE SOUND-PRESSURE LEVEL as measured in decibels is plotted as a function of time for a single vibrating piano string (*left*). Each division of the vertical scale corresponds to 10 decibels. At time *A* the key is depressed and the hammer strikes the strings, and at time *B* the key is released and the damper stops the motions of the strings. The plot can be resolved into two straight segments, of which the broken lines are extensions. The initial short-lived rapid decay constitutes the prompt sound, and the final long-lived slow decay, which gives the tone of the piano its characteristic singing quality, constitutes the aftersound. Noise in the electronic recording apparatus is responsible for jagged curves before *A* and after *B*. It turns out that when only one string of a unison group is vibrating, the vertical motion of the string (*middle*) gives rise to the prompt sound and the horizontal motion of the string (*right*) gives rise to the aftersound.



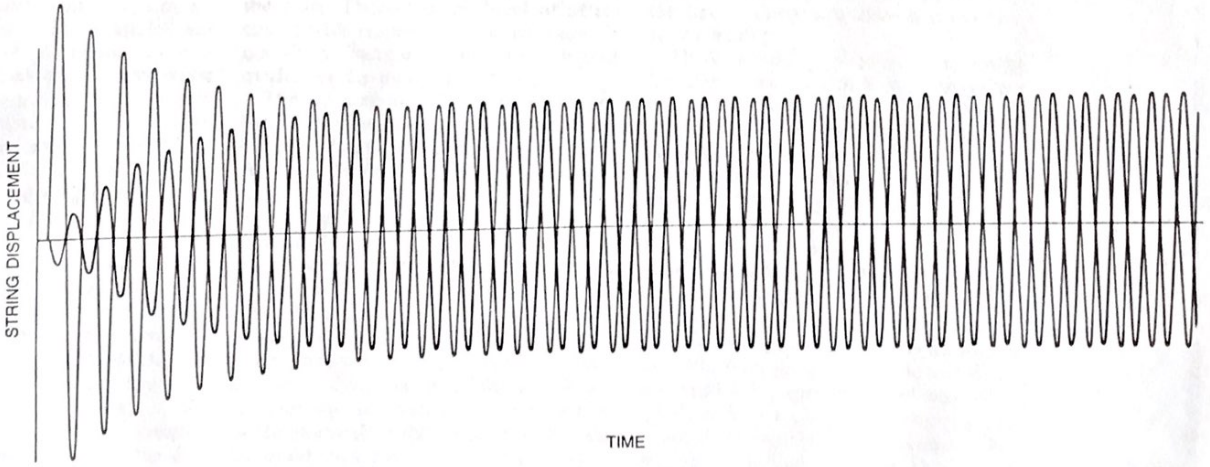
INTERFERENCE PATTERN between sound waves corresponding to vertical and horizontal vibrations of one string indicates that such waves emanate from different sources in the piano (*left*). This plot of relative sound pressure was made under the same conditions as the plot at the left in the upper illustration on this page except that the position of the recording microphone was changed. At this location sounds from the vertical and horizontal vibrations arrive with opposite phase, so that they cancel each other when their amplitudes are the same. When two strings in a unison group vibrate (*right*), the motion lasts much longer than when only one of them vibrates (*middle*).



FRICTIONAL FORCES that are proportionately larger at higher sound pressures are not responsible for the transition from prompt sound to aftersound. Such nonlinear frictional forces would cause the change to take place at the same sound pressure regardless of the string's initial displacement. In that case plots of sound-pressure level as a function of time made for various initial displacements would be horizontal translations of one another (*left*). It turns out, however, that the plots are vertical translations of one another (*right*), which is indicative of linear friction. Thus the prompt sound lasts the same amount of time regardless of the string's initial displacement.



HAMMER IMPERFECTIONS can result in string amplitudes that are not absolutely equal. Here two strings are set in motion at the same time but with the colored string having a larger amplitude than the black one. The motions of the strings start to decay, and when the amplitude of the black string approaches zero, the bridge continues to move because it is still being forced to do so by the colored string. As a result the black string not only reaches zero amplitude but also goes "beyond" it, building up a vibration of the opposite phase by absorbing energy from the bridge. Ultimately the motions are exactly antisymmetric. Such antisymmetric motion gives rise to aftersound.



UNA CORDA PEDAL, or soft pedal, increases the ratio of aftersound to prompt sound by shifting the entire keyboard so that a hammer strikes only one string of a pair. The unstruck string (*black*) starts to absorb energy from the bridge, which is vibrating synchronously with the other string (*colored*). The unstruck string immediately begins to move in phase opposite to the phase of the other string. As a result there is antisymmetric motion from the start, allowing the tone of the piano to retain its singing quality for quiet passages.

the motions of two strings are neither exactly the same (symmetric) nor exactly opposed (antisymmetric). In a piano the motions of the strings in one unison group will initially be almost perfectly symmetric, since the hammer apparently strikes the strings with the same strength at the same time. Minor imperfections in the hammer, however, will result in string amplitudes that are not absolutely equal. Consider, then, the case of two strings moving in phase but with the first string having a larger amplitude than the second one. At first both strings lose energy, and each string loses it faster than it would if it were vibrating alone, since the other one is "helping" the bridge to move.

When the amplitude of the second string (the one with the smaller amplitude) approaches zero, the bridge continues to move because it is being forced by the first string. As a result the second string not only reaches zero amplitude but also goes "beyond" it, building up a vibration of the opposite phase by absorbing energy from the bridge. Since the two strings are now moving in opposite phase, the bridge motion is less than it would be if one string were moving in the absence of the other. The two amplitudes asymptotically approach each other with opposite phase. Ultimately the motions of the two strings are exactly antisymmetric. It is the initial symmetric motion of the strings that constitutes prompt sound and the later antisymmetric motion of the strings that constitutes aftersound.

A useful way of looking at the situation I have just described is to think of the original motion as a superposition of two kinds of motion: a symmetric motion and an antisymmetric one. Let me illustrate what I mean by superposition. If I have 10 apples and you have six apples, we can describe the situation as a superposition of a symmetric state where we each have eight apples and an antisymmetric state where I have two apples and you have minus two apples. This is silly in the case of apples but useful in the case of vibrations, because it enables us to think of the symmetric component decaying at its characteristic rate, and the antisymmetric one decaying at a much lower rate (or, in the ideal case, not decaying at all). The algebraic sum of the two string amplitudes drops toward zero, but the difference remains constant for a long time.

Because the most general motion of two piano strings can be expressed as the superposition of symmetric motion and antisymmetric motion, the two kinds of motion are the normal modes of the piano-string system. It is interesting to note that the break in the decay of piano-string vibrations is not a unique phenomenon in physics. In fact, the decay characteristics are precisely analogous to those of such elementary subatomic particles as neutral kaons. Two kinds of kaon (K^0 and $\bar{K}^0$), a particle-antiparticle pair, can be formed through the strong interaction in nuclear collisions. In studying kaon decay physicists have identified two other varieties of kaon: K_S^0, which decays rapidly, and K_L^0, which decays slowly. It turns out that K_S^0 and K_L^0 are respectively symmetric and antisymmetric superpositions of K^0 and $\bar{K}^0$, just as the prompt sound and aftersound are respectively symmetric and antisymmetric superpositions of the motions of two piano strings. As a result a beam that initially consists only of K^0 particles will also have a kink in its decay curve.

The phenomenon of antisymmetric motion in a piano also accounts for the function of the una corda pedal, or soft pedal. The normal aftersound is about 20 decibels below the initial level of the prompt sound, a ratio that is apparently pleasing to the ear. This ratio, however, is not adequate for very quiet passages. When the piano is played softly, so that the amplitude of the prompt sound approaches the amplitude of the background noise in the concert hall, the aftersound becomes inaudible. If the notes are long, the piano will lose its sustaining quality and sound like a xylophone. To prevent this the piano is equipped with the una corda pedal, whose mechanical function is to shift the entire keyboard so that a hammer strikes only two strings of a unison triplet. Instead of exciting almost exclusively the symmetric motion with only a trace admixture of the antisymmetric motion, the una corda pedal excites both kinds of motion almost equally.

Why is this? The third string that was not hit by the hammer starts to absorb energy from the bridge, which is vibrating synchronously with the other two strings. The third string begins immediately to move in a phase opposite to the phase of the other two strings. As a result there is antisymmetric motion from the start. Therefore the level of aftersound with respect to prompt sound is markedly increased and the singing quality of the piano is restored.

The aftersound that comes either from antisymmetric motion or from horizontal polarization is quite soft compared with the prompt sound. And since such aftersound arises out of structural irregularities, it probably varies erratically from note to note. The mistuning of strings that constitute a unison group is a third mechanism that contributes to the aftersound. This mechanism is adjustable, however, and a skilled piano technician probably varies the mistuning to compensate for the erratic effects of the structural irregularities in order to equalize the strength of the aftersound from note to note. To analyze how mistuning affects the aftersound, a distinction must be made between this phenomenon and the phenomenon of "beats." If two independent oscillations whose frequencies differ slightly are added together, they will alternate slowly between a state of reinforcement (when they have the same phase) and a state of cancellation (when they have opposite phase). To the listener this sounds like a steady pitch with a pulsating loudness, which is what the word beats refers to. In a piano, however, the two strings do not vibrate independently. The motion of the bridge causes the vibration of one string to affect the vibration of the other. As a result not only the frequencies but also the decay rates are markedly affected.

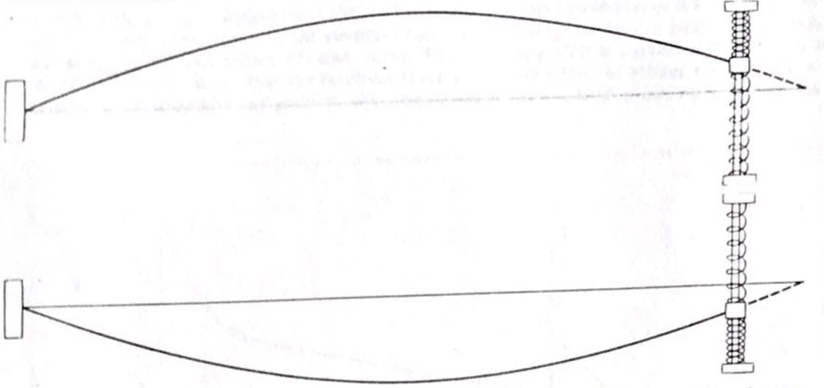
The mere motion of a support does not automatically lead to the dissipation of energy. In certain physical systems energy is not dissipated but is simply transferred back and forth between various subsystems. Consider a string attached to a ring that can slide up and down without friction on a fixed rod, and assume that the ring is sandwiched between two coil springs that act to keep the ring in its central position. When the string pulls up on the ring, the ring moves up, and when the string pulls down on the ring, the ring moves down. The motion of the support simulates the motion that would be executed by an extra piece of string attached in turn to a perfectly rigid support. Hence the effect of a "springy" support is to make the string move as if it were longer than it really is, and so to lower the frequency of the string.

A springy support does not, however, damp the motion of the string, because in the course of each complete cycle energy that flows into the support flows back into the string. As the string pulls the ring away from its central position against the force of the springs the string is doing work on the ring. On the other hand, as the ring returns to its central position assisted by the restoring force of the springs the ring is doing work on the string. Therefore there is no net energy transfer.

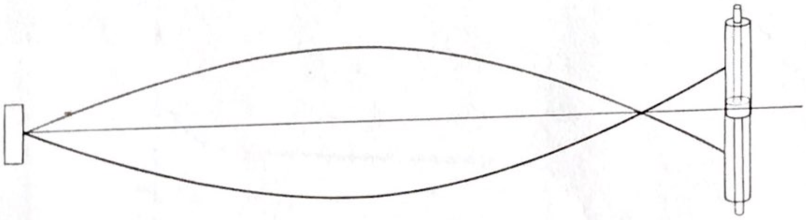
There would also be no net energy transfer in the case of a string attached to a massive block that can slide up and down without friction on a fixed rod. Here the motion of the block is governed not by a restoring force (since there are no coil springs) but by inertia. It is assumed for the sake of simplicity that gravity plays no role. Inertia acts to keep the block moving in whatever direction it is going. When the block reaches its maximum displacement in one direction, the string pulls back on it against inertia in order to slow it down and start it moving the other way. Inertia then propels the block through its central position to its maximum displacement in this direction. Once again the string pulls back on the block against inertia and sends it moving in the original direction, and the cycle continues to repeat itself.

The fact that the string is often pulling back on the block makes the string "think" it is shorter than it really is. Thus a massy support raises the frequency of the string. Like a springy support, a massy support does not damp the motion of the string. The work the string does on the block while pulling it back against inertia to reverse its direction is equal to the work the block does on the string while pulling the string along as inertia propels the block toward its central position.

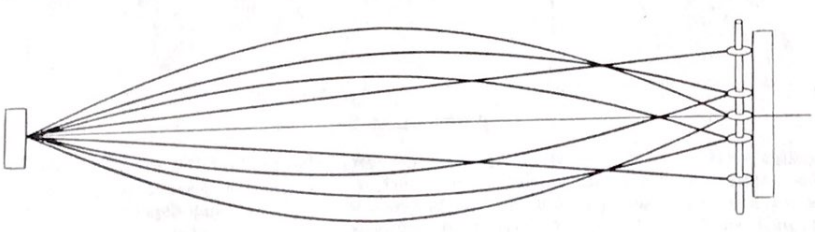
The idealized cases of a perfectly springy ring and a perfectly massy block indicate that supports can move without dissipating energy. Therefore the mere motion of the bridge in a piano does not indicate that the strings are losing energy. The bridge actually resembles a third idealized case: a "resistive" support where the phase difference between the displacement of the support and the force on it is a quarter of a cycle. In this case the frequency of the string remains the same but its motion is damped. An example of a perfectly resistive support is a ring whose motion is governed not by coil springs or by inertia but by friction. To overcome friction the string is constantly doing work on the ring, and so the string's energy is dissipated. That the phase shift is a quarter of a cycle is a shorthand way of saying that when the ring reaches its maximum displacement in either direction, the string reaches zero displacement (its central position), and when the string reaches a maximum displacement in either direction, the ring reaches zero displacement.



SPRINGY SUPPORT lowers the frequency of a string without damping the motion. An example of an ideal springy support is a ring sandwiched between two coil springs that slides up and down without friction on a fixed rod. The support lowers the string's frequency because it simulates motion that would be executed by an extra piece of string: the ring reaches maximum displacement when the string does and reaches zero displacement when the string does.



MASSY SUPPORT raises the frequency of a string without damping its motion. An example of an ideal massy support is a massive block that can slide up and down without friction on a fixed rod. Here the motion of the block is governed not by the restoring force of coil springs but by the effects of inertia. At the positions of the block shown here the string must pull back on the block against inertia in order to reverse its direction. The fact that the string is often pulling back on the massive block means the string "thinks" of itself as being shorter than it really is. As a result the ideal massy support has the effect of increasing the frequency of the string.



RESISTIVE SUPPORT leaves the frequency of the string undisturbed but damps its motion. An example of a perfectly resistive support is a ring that slides up and down on a rod but whose motion is retarded by friction against a wall. The string's motion is damped because the string is constantly doing work on the ring to overcome the friction between the ring and the wall. The phase difference between the displacement of the support and the force on it is a quarter of a cycle: when the ring reaches a maximum displacement, the string reaches zero displacement; when the string reaches a maximum displacement, the ring reaches zero displacement.

Let me explain how the ideal resistive situation exhibits the characteristics of the aftersound of mistuned piano strings. What happens if two strings are started in exactly antisymmetric motion but with frequencies that are not quite identical? At first the support is not moving, since the initial string motion is antisymmetric. The string with the higher natural frequency begins to advance in phase over the other string, and so their motion is no longer purely antisymmetric. As a result the strings exert a small force on the bridge. The phase difference between the force and the motion of each string is indeed a quarter of a cycle [*see top illustration on page 126*]. The strings reach points of maximum displacement when the force is smallest and points of minimum displacement when the force is largest. The former relation holds because at the points of maximum displacement the strings' amplitudes are opposite, and so they cancel each other to produce the smallest force on the bridge. The latter relation holds because at the points of minimum displacement the strings' amplitudes are of the same sign, and so they add together to exert the greatest force on the bridge.

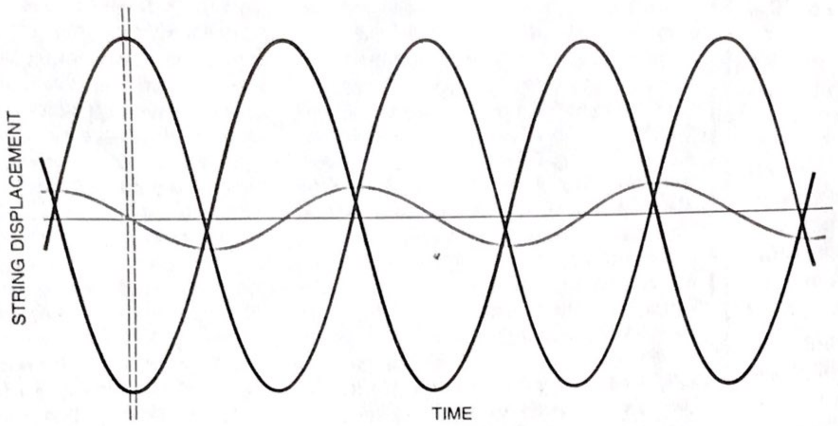
On the assumption that the bridge is a purely resistive support the bridge develops in turn a small motion that is a quarter of a cycle out of phase with the force. Hence the motion of the bridge is in phase with the motion of one of the strings and in opposite phase with the motion of the other. The in-phase string "sees" the bridge as a springy support, whereas the opposite-phase string sees the bridge as a massy support. This means that the frequency of the in-phase string is raised and the frequency of the opposite-phase string is lowered. It turns out that the string with the lower original frequency will have its frequency raised and the other string will have its frequency lowered, so that both end up vibrating at precisely the same frequency. The decay rate, however, is no longer zero, as it was for pure antisymmetric motion where the strings vibrate at exactly the same frequency. In other words, the mistuning generates a sound of a single frequency that decays slowly.

If two strings are started with perfectly symmetric motion but with frequencies that are not quite identical, one string will begin to fall behind the other in phase. As a result the bridge motion will not be exactly a quarter of a cycle out of phase with the motion of either string, as it would be if the strings had continued to move in a perfectly symmetric fashion. The frequency of each string is raised or lowered, depending on whether the phase difference between the motion of the string and the motion of the bridge is closer to the phase difference that characterizes a springy support or closer to the phase difference that characterizes a massy support. Once again no beat is heard as the frequencies are brought together. Since the slight mistuning introduces a trace of antisymmetric motion, the damping is a little smaller than it is in the perfectly symmetric case where the precisely tuned strings cooperate fully in moving the bridge.

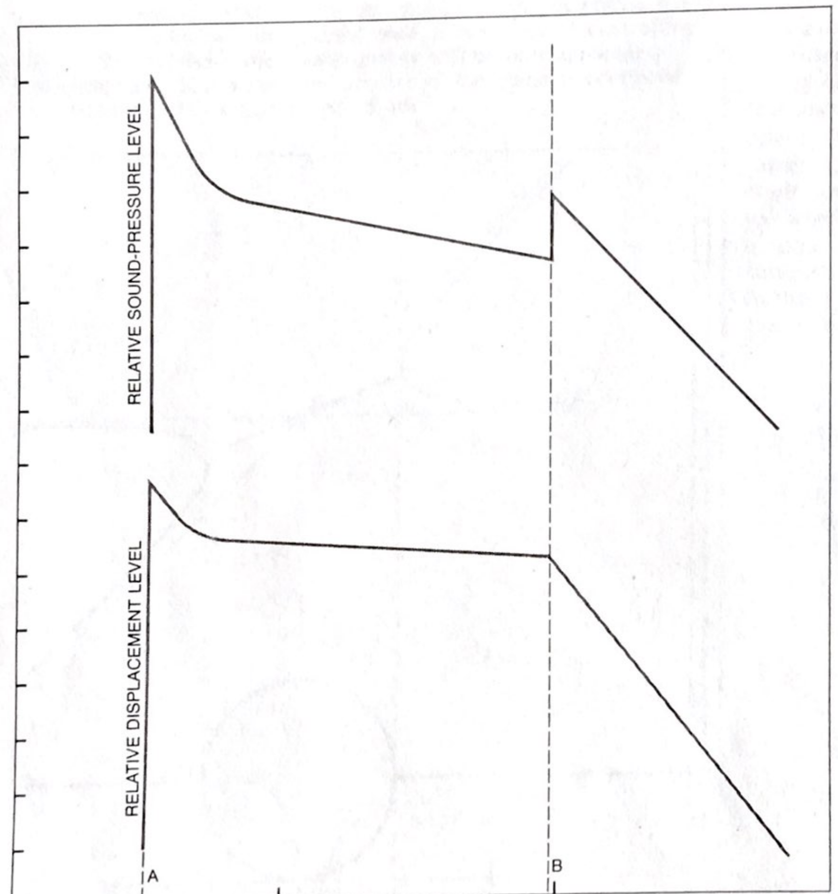
In the cases of initial symmetric motion and initial antisymmetric motion the presence of resistive coupling tends to lock together the frequencies of the two strings but to alter the decay rates. Of course, there is a limit to how far apart the original frequencies can be. As the mistuning is increased the phase difference between the strings increases until it reaches a quarter of a cycle, where the frequencies break apart. At even greater phase differences beats are heard, and the decay rates of both the symmetric motion and the antisymmetric motion become equal to the decay rates for uncoupled strings.

When two strings are in tune, their motion can always be expressed as the superposition of symmetric and antisymmetric modes. When two strings are slightly mistuned, their motion can still be expressed as the superposition of two modes: an almost antisymmetric mode whose damping is small, although not quite zero, and an almost symmetric mode whose damping is large, although not twice as large as the single-string rate. In both modes the amplitudes of the two strings are equal. If a hammer strikes the strings at the same time and with the same strength, the exactly symmetric motion the hammer excites is not a normal mode; rather, it must be viewed as a superposition consisting mostly of the rapidly decaying (almost symmetric) mode but containing an admixture of the long-lived (almost antisymmetric) mode as well. The amount of this admixture depends on how different these normal modes are from perfect symmetry and perfect antisymmetry, which depends in turn on the extent of the mistuning.

The major difference between the contribution to the aftersound due to the mistuning and the contributions due to the horizontal polarization and the antisymmetric motion is that a skilled piano tuner can adjust the former but not the latter. I think this explains seemingly random variations in unison tuning that were observed by Roger E. Kirk of the D. H. Baldwin Company. A skilled piano tuner varies the mistuning in such a way as to make the aftersound uniform and smooth from note to note by compensating for the irregular effects of the



RESULTANT FORCE (*colored curve*) on the bridge of a piano is proportional to the algebraic sum of the strings' displacements. When the motions of the two strings (*black curves*) are almost perfectly antisymmetric, the resultant force on the bridge is about a quarter of a cycle out of phase with the motion of either string. That the phase shift is a quarter of a cycle is a shorthand way of saying that the resultant force is smallest when the strings reach points of maximum displacement and greatest when the strings reach points of minimum displacement.

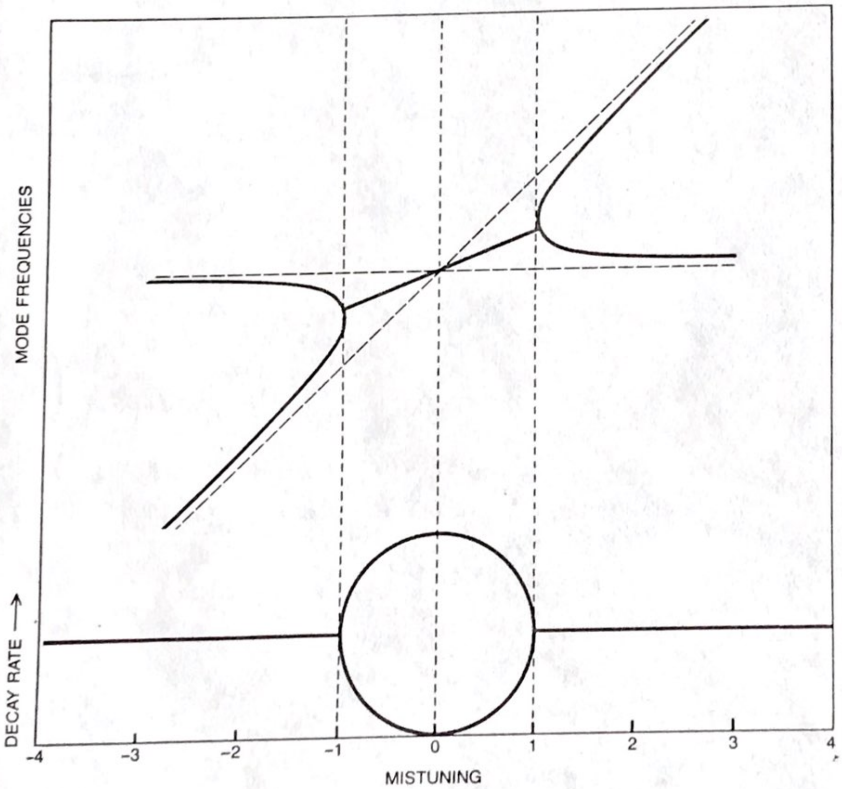


KNOWLEDGE OF ANTISYMMETRIC MOTION makes it possible to construct a piano that could introduce an accent into the middle of an otherwise sustained note. The piano could be made with split dampers that would separately stop the motion of each string in a unison group. At time *A* a hammer sets in motion the two strings and at time *B* one of the strings is damped. The top plot shows the relative sound-pressure level as a function of time, and the bottom plot shows the relative displacement level of the undamped string. From *A* to *B* the curves are typical ones that decay rapidly at first and slowly at the end. When one of the strings stops moving at *B*, the strong antisymmetric motion of the two strings abruptly stops and the undamped string immediately starts to decay at the original rapid rate. The sound pressure experiences a sudden increase because there is scarcely any antisymmetric motion to retard the motion of the bridge. The sudden increase gives rise to an accent in the middle of the note.

horizontal polarization and the antisymmetric motion. In this way the piano attains its characteristic beauty of tone that less skilled piano tuners are unable to induce. To test my hypothesis I would have one tuner tune the same piano a number of times, with someone else detuning it in between. If the same "random" mistunings manifested themselves each time, it would prove that the mistunings were not random at all.

Piano physics has now reached the stage where each step forward raises more questions than it answers. For the investigator this is an extremely exciting stage. The trial-and-error method that has historically characterized the development of musical instruments is particularly inefficient for such a huge acoustical structure as a piano, where the investment required for a new design is so large that it discourages experimentation. For this reason the emergence of a detailed physical picture of the workings of the piano promises to have a tremendous impact on piano technology. Even the present incomplete picture suggests innovations. For example, the understanding of antisymmetric motion points to the construction of a piano that could introduce an accent into the middle of an otherwise sustained note.

Picture a piano with split dampers that could separately stop each string in a unison group. Perhaps a special pedal would control the split dampers. Now consider a unison group of two strings. When the corresponding key is depressed, a note is heard that has a typical mixture of prompt sound and aftersound. After a few seconds the symmetric component of the motion has completely died away and only aftersound can be heard. At this point the special pedal is depressed that damps the motion of one of the strings. As a result the strong antisymmetric motion of the two strings abruptly stops and the undamped string immediately starts to decay at the original rapid rate. The sound pressure suddenly increases, as the amplitude of the motion of the bridge soars in the absence of the retarding effect of the antisymmetric motion of the strings. In this way a sharp accent is introduced into the middle of the otherwise sustained note. Other ways of controlling tone quality will become apparent once the physics of the piano is completely worked out.



FREQUENCIES OF A PAIR OF STRINGS lock together when the motions of the strings are coupled through a purely resistive support. The mistuning, or difference between the uncoupled frequencies, is given in "natural units," which are related to the single-string damping rate. For a typical pair of strings in the middle of the keyboard, one natural unit is about a third of a vibration per second. The broken lines in the top graph indicate the frequencies in the absence of coupling. The point where the broken lines cross each other is where the two strings have exactly the same frequency. In a piano the presence of a purely resistive support causes frequencies with a mistuning of either +1 or −1 natural unit to come together and lock at a common frequency. For smaller mistunings the frequencies stay locked but the decay rate, which equals the single-string rate for larger mistunings, splits for the two strings (*bottom*).